SOLUTION OF SHORT QUESTIONS

Short Questions

Write the short answers of the following Questions:

Q.1: Write the Phasor (Vectors) $z = a + jb$ in Trigonometric and exponential forms. (IA-2019)

Ans. $z = a + jb$

Trigonometric OR Polar Form

$$\boxed{z = r(\cos\theta + j\sin\theta) = r\angle\theta}$$

Exponential form

$$\boxed{z = re^{j\theta}}$$

Q.2: Express $\sqrt{2}\ \angle 45^\circ$ in rectangular form. (IIA-2018)

Sol. $\sqrt{2}\ \angle 45^\circ$

$$= \sqrt{2}(\cos 45^\circ + j\sin 45^\circ)$$

$$= \sqrt{2}\left(\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right) = \sqrt{2}\left(\frac{1+j}{\sqrt{2}}\right) = \boxed{1+j}$$

Q.3: Express $\sqrt{3} + j$ in polar form. (IA-2016), (IA-2018), (IA-2022)

Sol. Let $Z = \sqrt{3} + j$ Here $a = \sqrt{3}$ & $b = 1$

$$r = \sqrt{a^2 + b^2}$$

$$r = \sqrt{(\sqrt{3})^2 + (1)^2}$$

$$r = \sqrt{3+1} = \sqrt{4} = 2$$

$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$

$$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30^\circ$$

$$z = r\angle\theta = \boxed{2\ \angle 30^\circ}$$

Q.4: Express $z = e^{j\frac{\pi}{3}}$ in rectangular form (i.e, $a + jb$)

Sol. $Z = e^{j\frac{\pi}{3}} = \cos\frac{\pi}{3} + j\sin\frac{\pi}{3} = \cos 60^\circ + j\sin 60^\circ = \boxed{\frac{1}{2} + j\frac{\sqrt{3}}{2}}$

SOLUTION OF SHORT QUESTIONS

Q.5: Find the product of $Z_1 = 2\angle 15^\circ$, $Z_2 = -1\angle 30^\circ$ (IIA-2018)

Sol. $Z_1Z_2 = (2\angle 15^\circ)(-1\angle 30^\circ) = (2)(-1)\angle(15^\circ + 30^\circ) = \boxed{-2\angle 45^\circ}$

Q.6: Given that $Z_1 = 4\angle 60^\circ$, $Z_2 = 2\angle 30^\circ$ find $\frac{Z_1}{Z_2}$. (IA-2019)

Sol. $\frac{Z_1}{Z_2} = \frac{4\angle 60^\circ}{2\angle 30^\circ} = 2\angle(60^\circ - 30^\circ) = \boxed{2\angle 30^\circ}$

Q.7: If $A = 2 + j3$, and $B = 8 + j5$, then find $A + B$.

Sol. $A + B = 2 + j3 + 8 + j5 = \boxed{10 + j8}$

Q.8: Simplify $(2 + j3)(4 - j2)$.

Sol. $(2 + j3)(4 - j2) = 8 - j4 + j12 - j^2 6 = 8 + j8 + 6 = \boxed{14 + j8}$

Q.9: If $A = 20\angle 60^\circ$ and $B = 5\angle 30^\circ$, then find AB.

Sol. $AB = (20\angle 60^\circ)(5\angle 30^\circ) = (20)(5)\angle(60^\circ + 30^\circ) = \boxed{100\angle 90^\circ}$

Q.10: Simplify $\frac{(5\angle 45^\circ)(6\angle 60^\circ)}{3\angle 30^\circ}$

Sol. $\frac{(5\angle 45^\circ)(6\angle 60^\circ)}{3\angle 30^\circ} = \frac{(5)(6)\angle(45^\circ + 60^\circ)}{3\angle 30^\circ} = \frac{30\angle 105^\circ}{3\angle 30^\circ}$

$= 10\angle(105^\circ - 30^\circ) = \boxed{10\angle 75^\circ}$

Q.11: Write the conjugate and modulus of $-2 + j$. (IA-2022)

Sol. Conjugate $= \overline{-2 + j} = \boxed{-2 - j}$

Modulus $= \sqrt{(-2)^2 + (1)^2} = \sqrt{4 + 1} = \boxed{\sqrt{5}}$

Q.12: Write the conjugate and Modulus of $-\frac{2}{3} - j\frac{4}{9}$. (IIA-2016)

Sol. Conjugate $= \overline{-\frac{2}{3} - j\frac{4}{9}} = \boxed{-\frac{2}{3} + j\frac{4}{9}}$

Modulus $= \sqrt{\left(\frac{-2}{3}\right)^2 + \left(-\frac{4}{9}\right)^2} = \sqrt{\frac{4}{9} + \frac{16}{81}} = \sqrt{\frac{36 + 16}{81}} = \boxed{\frac{\sqrt{52}}{9}}$

SOLUTION OF SHORT QUESTIONS

Simplify the phasors (vectors) and write the results in rectangular form:

Q.13: $(7-j2)-(4+j5)$

Sol. $(7-j2)-(4+j5)$

$= 7 - j2 - 4 - j5$

$= \boxed{3 - j7}$

Q.14: $(-5+j3)(2-j3)$

Sol. $(-5+j3)(2-j3)$

$= -10 + j15 + j6 - j^2 9$

$= -10 + j21 + 9 = \boxed{-1 + j21}$

Q.15: $\dfrac{-9+j4}{8-j3}$

(IA-2016) (IA-2017)

Sol. $\dfrac{-9+j4}{8-j3}$

$= \dfrac{-9+j4}{8-j3} \times \dfrac{8+j3}{8+j3}$

$= \dfrac{-72-j27+j32+j^2 12}{(8)^2-(j3)^2}$

$= \dfrac{-72+j5-12}{64+9}$

$= \dfrac{-84+j5}{73}$

$= \boxed{\dfrac{-84}{73} + j\dfrac{5}{73}}$

Q.16: $\dfrac{1}{4-j5} - \dfrac{1}{5-j4}$

(IIA-2016), (IA-2018)

Sol. $\dfrac{1}{4-j5} - \dfrac{1}{5-j4}$

$= \dfrac{1(5-j4)-1(4-j5)}{(4-j5)(5-j4)}$

$= \dfrac{5-j4-4+j5}{20-j16-j25+j^2 20}$

$= \dfrac{1+j}{20-j41-20} = \dfrac{1+j}{-j41}$

$= \dfrac{1+j}{-j41} \times \dfrac{j41}{j41} = \dfrac{j41+j^2 41}{-j^2 41^2}$

$= \dfrac{j41-41}{1681} = \dfrac{-41+j41}{1681}$

$= \dfrac{41(-1+j)}{1681}$

$= \dfrac{-1+j}{41} = \boxed{-\dfrac{1}{41} + j\dfrac{1}{41}}$